Exam. Code : 211004
Subject Code : 4913

M.Sc. (Mathematics) 4th Semester (Batch 2020-22)

MATH-575 : DISCRETE MATHEMATICS—I

Time Allowed—3 Hours] [Maximum Marks—100

Note :— Attempt *five* questions in all, selecting at least *one* question from each section. The fifth question may be attempted from any section. All questions carry equal marks.

SECTION—A

1. (a) Draw the Hasse diagram of the relation $\subseteq$ on P(A), where A = {a, b, c}.

   (b) Define relation and equivalence relation. The relation $R \subseteq N \times N$ is designed by $(a, b) \in R$ if and only if 5 divides $b - a$. Show that R is an equivalence relation.

2. (a) How many people among 200000 people are born at same time (hour, minute, seconds) ? Use Pigeonhole principle to find it.

   (b) In a certain examination, 53% students pass in Economics, 61% in Politics, 60% in History, 24% in Economics and Politics, 35% in Politics and History, 27% in Economics and History and 5% pass in none these subjects. How many students passed in all the three subjects ?

SECTION—B

3. (a) Write the negation of each of the following disjunction :

   (i) Ram is in class XI or Arun is in Class XII.

   (ii) 9 is greater than 4 or 6 is less than 8.

   (b) Prove that $(p \wedge q) \rightarrow (p \wedge q)$ is tautology but $(p \vee q) \rightarrow (p \wedge q)$ is not.

4. Show that $p \leftrightarrow q \equiv (p \vee q) \rightarrow (p \wedge q)$, using :

   (a) Truth table

   (b) Algebra of propositions.

SECTION—C

5. State and prove fundamental theorem of semi group homomorphism.

6. Define monoids with examples. Let $(G, *)$ and (G', o) be monoids with identities e and i respectively. Let $f : G \rightarrow G'$ be a homomorphism from $(G, *)$ onto (G', o) then $f(e) = i$.

SECTION—D

7. (a) Solve the following $y_{n+2} - y_{n+1} - 2y_n = n^2$.

   (b) Find particular solution of $S_n + S_{n-1} = 3n.\ 2^n$.

8. (a) Find the generating function which will give the number of integral solution of $x+y+z = 5$, if :

   (i) Each variable is at least 2 and at most 4

   (ii) $0 \leq x \leq 5, 2 \leq y \leq 6, 5 \leq z \leq 8$, x is even and y is odd.

   (b) Use generating functions to solve the recurrence relation :

$$a_r - 9a_{r-1} + 20a_{r-2} = 0,\ a_0 = -3,\ a_1 = -10$$